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**EFFORTS IN MARKOV MODELING OF  
WEATHER DURATIONS**

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**Final Report  
1 October 1979 - 30 September 1981**

**15 February 1982**

**Approved for public release; distribution unlimited**

**AIR FORCE GEOPHYSICS LABORATORY  
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A portable version of the algorithm for finding the maximum of Gaussian-Markov weather variates has been obtained employing cubic B-splines. Efforts to model and predict equipment response to temperature modified by the thermal lag of the equipment are described.		

## FINAL REPORT

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In the previous contractual period, the basic objective was the distribution of the maximum of the stationary Gaussian-Markov process over an interval of specified duration. This objective was achieved and presented in the Final Report [1], "The Maximum of the Stationary Gaussian-Markov Process Over an Interval of Specified Duration." (AFGL-TR-79-0282).

Even though the desired distribution was available with great accuracy, an improved portable version of the software was requested as a simple operational tool to be used for mission planning. The portable version requested was provided [2] in Scientific Report No. 1 (AFGL-TR-81-0068), "An Algorithm for  $F_0(y)$  Using Cubic B-splines", and the software needed delivered to AFGL. The B-spline approach encapsulates the basic two-parameter tables into a simpler representation form approximating the tables with great accuracy. This portable form requires only a microcomputer and gives very quick response times.

The next problem addressed was the following: Consider the temperature at a specified geographical location and specified time of year. An item of military equipment such as a bulldozer has a substantial thermal inertia which "sees" the average value of the temperature over a given period rather than the instantaneous temperature. The following question then arises. Suppose the temperature is modeled by a stationary Gaussian-Markov process  $X(t)$ , and suppose that the average temperature  $Y_\theta(t)$  is an exponentially weighted average of  $X(t)$ , i.e.,

$$(1) \quad Y_\theta(t) = \int_{-\infty}^t \theta e^{-\theta(t-t')} X(t') dt' .$$

Here  $\theta$  is the reciprocal of the mean response time to  $X(t)$ . Such exponential weighting represents fairly the thermal lag response of equipment. Then  $Y_\theta(t)$  is also stationary, and it is easy to show that  $EY_\theta(t) = E[X(t)]$ . The long-term average equipment temperature is therefore independent of the thermal mass of the equipment.

On the other hand, the variability of the equipment temperature is reduced by the averaging and the greater the thermal inertia, the smaller is the variability. The distribution of  $Y_\theta(t)$  is known to be normal, and its variance is readily available. One can show that

$$(2) \quad \text{Var}[Y_\theta] = \text{Var}[X] \frac{1}{1 + \frac{\tau_\theta}{\tau_{\text{REL}}}}$$

where  $\tau_\theta = \frac{1}{\theta}$  is the mean time to thermal equilibrium of the equipment and  $\tau_{\text{REL}}$  is the relaxation time of the process  $X(t)$  describing the temperature at the site of interest (cf. [1]). It is clear that

$$(3) \quad \text{Var}[Y_\theta] < \text{Var } X$$

as expected. Of considerable meteorological interest is the distribution of the maximum of  $Y_\theta(t)$ , i.e., of

$$(4) \quad M_\theta(t) = \max_{t \leq t' \leq t+\Delta} Y_\theta(t'),$$

the counterpart with averaging of the previous maximum obtained, for which  $\tau_{\text{REL}} = 0$ .

To address such a problem one must examine the time-dependent behavior of the joint distribution of  $X(t)$ ,  $Y_\theta(t)$  and, in particular, one must solve the following first passage problem related thereto. One requires, as in [1], the ergodic exit time from the subset  $S = \{-\infty < x < \infty, -\infty < y < Y\}$  of the state space  $R_2$  of the joint process  $X(t)$ ,  $Y(t)$ . This is a very difficult problem, for which, ideally, an analytical solution is needed as a basis for algorithmic evaluation. Despite great effort, however, such a solution has not been found. It has therefore been necessary to consider processes which approximate the desired processes  $Y_\theta(t)$  and  $M_\theta(t)$ , and which lend themselves to algorithmic methods.

Two papers were written with this objective in mind. Both [3,4] have been submitted as Scientific Reports. The first, "Row-continuous finite Markov chains, structure and algorithms," considers a bivariate Markov chain  $[J(t), N(t)]$  on a rectangular lattice of integers  $N = \{(j,n); 0 \leq j \leq J, 0 \leq n \leq M\}$  which has the property of row-continuity, i.e., is such that  $M(t)$  changes by at most one in any transition. Algorithmic procedures are developed to find the ergodic distribution on the chain and expected sojourn times and ergodic exit times on subsets of state space are of interest.

The second paper, "The Ehrenfest chain as an approximation to the O-U process," develops a truncated birth-death process  $N(t)$  which approximates the desired process. The hope is to employ this Ehrenfest chain approximation as the marginal process  $J(t)$  of a bivariate chain  $[J(t), M(t)]$  in which  $M(t)$  would mirror the behavior of the desired  $M_\theta(t)$ .

The effort continues as a dissertation study of a graduate student, even though the contract has been terminated. The algorithms provided by the row-continuous approach are burdened by the need for large values of J and M to have any accuracy. These large J, M values in turn lead to long computer run times and the feasibility of the algorithmic approach and its accuracy must be established.

*Julian Keilson*  
Julian Keilson  
Principal Investigator

Date: February 15, 1982



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- [1] Keilson, J. and H. F. Ross, "Gaussian Markov Related Variates for Meteorological Planning," AFGL-TR-79-0282, November 1979.
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